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Effect of Frequency-Averaging on Estimation of Clutter Statistics Used in Setting CFAR Detection Thresholds

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FOR THE COMMANDER

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EFFECT OF FREQUENCY-AVERAGING ON ESTIMATION OF CLUTTER STATISTICS USED IN SETTING CFAR DETECTION THRESHOLDS

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ABSTRACT

Incoherent averaging of radar returns measured at different frequencies has been shown to be an effective method of improving fixed target detection performance. This improvement results from the fact that frequency-averaging narrows the amplitude probability distributions of both the target and clutter returns (after averaging), thereby allowing returns from targets to the more easily distinguished from those of clutter.

This report examines one aspect of this improvement - the effect of frequency averaging on decreasing the sample variance (i.e., narrowing the distribution) of the measured clutter returns. The attainable reduction in clutter sample variance that results from frequency-averaging is shown to be strongly dependent upon the spatial "non-unformity" of the clutter, as characterized by the variance of its intrinsic spatial amplitude distribution. The more non-uniform the clutter, the less effective frequency-averaging will be in reducing the clutter sample variance and corresponding CFAR threshold setting. An analytic expression is derived for the amount of sample variance reduction provided by frequency averaging, as a function of clutter spatial standard deviation and the number of frequencies averaged. The amount of variance reduction predicted by this expression is comfirmed by experimental measurements (at Ku band) of clutter returns, at 16 and 32 different frequencies, for a variety of different clutter types. The separation between the frequencies at which these measurements were made was on the order of the radar bandwidth, thereby ensuring that the returns measured at those frequencies were statistically independent of one another.

For most commonly encountered types of ground clutter, it is found that averaging clutter returns over four different frequencies reduces the clutter sample standard deviation, and the corresponding CFAR threshold offset, by 35% to 50% relative to that required for data measured at a single frequency (no averaging). However, little additional reduction in threshold level appears to be obtained when the returns are averaged over more than 8 frequencies.

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1.0 INTRODUCTION

The detection of fixed ground targets in clutter is based upon the amplitude of the return from a target exceeding that from the surrounding ground clutter. The detection process used is that of the CFAR (Constant False Alarm Rate) detector, in which the amplitude of each resolution cell is compared against a threshold. This threshold is set at a specified level above the average amplitude of the clutter in the vicinity of the resolution cell being tested for the presence of a target return. If the amplitude of the return in the resolution cell being tested exceeds this threshold, a target is declared to exist in that cell.

However the amplitude of the clutter returns is usually randomly distributed over both space and frequency. Consequently, if the resolution cell being tested contains only clutter (i.e., does not contain a target), the specified threshold level may still be exceeded, resulting in a false target report, or false alarm. Clearly, the higher the setting of the threshold, the lower the false alarm rate, but also the lower will be the probability of detecting any given target. However, because the number of resolution cells to be tested in a given radar scene usually greatly exceeds the expected number of targets in that scene, it is important to set the threshold high enough to keep the corresponding false alarm rate at very low value - usually on the order of 10^{-3} or less.

The distribution of clutter returns with respect to spatial location has been shown to be log-normal in nature for many different kinds of commonly encountered ground clutter [2],[3]. This distribution is a two-parameter distribution, and hence is uniquely characterized by both a mean value

and a variance. The experimental results presented in Chapter 5 indicate that the log-normal spatial amplitude distribution for any specified type of ground clutter (i.e., level fields, scrub, dirt, deciduous forest, conifer forest, etc.) can be uniquely specified by these two parameters. However, it should be noted that, up to the present time, most quantitative characterizations of clutter have been in terms of only a single parameter, σ_o, the mean value of clutter reflectivity per unit area. The mean value of the clutter return received by a radar is proportional to $A\sigma_o$, where A is the projected area of the radar's range-azimuth resolution cell on the clutter surface. However the value of σ_o for a given clutter type generally gives no indication of the spatial non-uniformity of the clutter. This latter quantity is characterized by the variance (or equivalently the standard deviation) of the spatial clutter amplitude distribution. In this report this spatial variance has been given the designation σ_c^2. As will be seen, it is usually convenient to work in terms of the distribution of the logarithm of clutter amplitudes. This has the fortuitous consequences that 1) the resultant distribution is normal (for log-normally distributed clutter) and 2) σ_c can be expressed in units of dB.

Therefore, to accurately characterize the spatial amplitude probability distribution of a given clutter type, both the mean reflectivity σ_o and the spatial standard deviation, σ_c, must be specified. Although many measurements of σ_o have previously been reported in the literature [4], [8], measurements of σ_c do not appear to be as common. However, the performance of an amplitude CFAR detector in clutter depends upon both of these parameters, [1], particularly σ_c. The data presented in Chapter 5

indicates that for many common types of homogenous ground clutter σ_c usually ranges between 0.2 and 2.5 dB.

In the context of this report homogenous clutter is defined as clutter that is all from the same statistical population (i.e., from a region of all grass, all trees, all scrub, etc.), and hence is characterized by a unique set of the parameters: (σ_o, σ_c). Note that this definition does not require that the clutter be spatially uniform. Uniform clutter is defined as clutter for which σ_c=0, and is therefore a special case of homogenous clutter. Examples of uniform clutter might be an airport runway, an empty parking lot, a level mown golf course, etc. It should be noted however that radar returns from uniform clutter will, in general, still be randomly distributed with respect to radar frequency. Finally non-homogenous clutter is defined as mixtures of different kinds of homogenous clutter within a local area.

In addition to being randomly distributed, ground clutter is usually also highly non-homogenous over a given radar surveillance area. For example, a typical area may include patches of forest, fields, roads, rocky hillsides, etc., and the statistics of the clutter distributions for each of these patches will be quite different - i.e., the statistics (average value and variance) of the distribution of the returns from a level, mown field will generally be much smaller than that for returns from tree cover or from rough terrain. Consequently, if the false alarm rate is to be maintained constant over a radar surveillance area that contains a variety of clutter types such as forest and fields, the CFAR threshold must be adaptively set

with respect to the statistics of the local clutter in the vicinity of each resolution cell being tested.

1.1 Computation of CFAR Detection Threshold

Ideally, if the mean value, μ_x, and variance, σ_x^2, of the measured clutter returns were known exactly, for each local region of the radar surveillance area, it would be possible to compute the threshold setting required to maintain a constant false alarm rate in that region as follows:

$$T = \mu_x + k\sigma_x$$

where μ_x and σ_x are the mean and standard deviation of the local clutter returns processed by the CFAR detector. The factor k in the above expression is called the threshold multiplier and is uniquely determined by the specified value of clutter false-alarm rate, P_{fa}. For typical false alarm rates between 10^{-2} to 10^{-5}, the value of k ranges between about 3 and 5.

The term $k\sigma_x$ is of special significance since it effectively represents the threshold offset relative to the mean clutter level. When frequency-averaging is used, μ_x and σ_x are the mean and standard deviations of the frequency averages of the clutter returns. σ_x is shown to be a function of both the intrinsic spatial non-uniformity of the local clutter, i.e., σ_c, and the number of frequencies over which the clutter returns are averaged. σ_x decreases at a rate somewhat less than the

square-root of the number of frequencies over which the returns are averaged, asymptotically approaching the value σ_c in the limit. As mentioned previously, σ_c is equal to the standard deviation of the intrinsic spatial distribution of mean clutter amplitudes (averaged over frequency), and is a function of clutter type. Thus the value of σ_c for tree cover or rocky terrain will be larger than that for asphalt pavement or a mown field.

For reasons to be given later, it is convenient to express the clutter statistics μ_x, σ_x and σ_c, and the threshold level T in terms of dB. In terms of these units, the value of σ_c will range between about 0.5 and 3 dB for the different types of ground clutter encountered in a typical radar scene. The corresponding values of σ_x will range from about 1.0 dB to 7.0 dB, depending upon the number of frequencies averaged. As mentioned earlier, typical values of the threshold multiplier, k, range between 3 and 5, depending upon the specified value of false alarm rate. Consequently, the resulting threshold offset, $k\sigma_x$, will usually range between a low of about 2.0 dB to a high of about 30 dB depending upon local clutter conditions, the number of frequencies averaged, and the specified value of false alarm rate P_{fa}. The knowledge of the clutter standard deviation, σ_x, is therefore especially critical in computing the required threshold setting.

In practice however, values of μ_x and σ_x of the local clutter will never be known exactly, but must be estimated from the sample statistics of returns from the local clutter in the region being tested. As mentioned above, because of the different types of clutter that can appear

in a typical radar surveillance area, these local sample statistics may vary widely over a given radar scene. Hence the actual threshold setting, T, used for testing the return from a given cell is computed as a function of the sample mean, $\overline{X}$, and sample variance, V_x, of the frequency averages of returns from the local clutter in the neighborhood of the cell being tested.

$$T = X + k\sqrt{V_x} \tag{1.2}$$

Since the sample mean, $\overline{X}$, and sample standard deviation, $\sqrt{V_x}$, are themselves random variables - T will also be a random variable rather than just a deterministic function of the threshold multiplier k. In effect $\overline{X}$ and V_x are merely estimates of the (unknown) values of the actual mean and variance of the local clutter, μ_x and σ_x^2.

1.2 <u>Effect of Frequency Averaging</u>

As can be seen from equation 1.2, the computed threshold setting is critically dependent upon the clutter sample variance V_x. It is shown herein that if $\overline{X}$ and V_x are computed from <u>frequency-averages</u> of the clutter samples, rather than from clutter samples measured at only a single frequency, the expected value of $\overline{X}$ remains unchanged, but that of V_x will decrease. This is an important result since it means that frequency-averaging of the returns prior to performing the CFAR test lowers the threshold setting required to maintain a given value of false alarm rate (fixed k), with the result that target detectability is improved.

The reason for this phenomenon is as follows. The amplitude of clutter returns varies randomly with respect to both space and frequency. Specifically, the amplitude of the clutter return from any given resolution

cell will be a random function of frequency. The mean value, averaged over frequency, of the returns from that cell then represents the intrinsic amplitude of the clutter in that cell. However these mean values with respect to frequency are in turn random variables with respect to spatial location (in range and azimuth), and are usually log-normally distributed over space. Frequency-averaging reduces that component of variability of the clutter returns due to frequency. As a result, the expected value of the sample variance, V_x, that is computed from frequency-averages of clutter measurements, decreases toward a lower bound equal to the intrinsic spatial variance of the clutter, σ_c^2, as the number of frequencies over which the clutter returns are averaged is increased. The more uniform the clutter, the more effective frequency-averaging will be in reducing V_x.

The effect of frequency-averaging on the computation of the clutter sample statistics and the resulting setting of the CFAR detection threshold is the subject of this report. Of particular interest is the effect on the sample standard deviation $\sqrt{V_x}$, since this directly affects the required level of the CFAR detection threshold. It is shown that the expected value of $\sqrt{V_x}$ for single frequency clutter samples ranges from about 5.5 dB to 6.5 dB for most common types of ground clutter. As the number of frequencies over which the clutter returns are averaged increases, the expected value of $\sqrt{V_x}$ approaches the value of the standard deviation of the clutter's underlying spatial distribution σ_c - a value that is generally

between about 0 and 3 dB, depending upon clutter type. Furthermore, most of the attainable reduction in $\sqrt{V_x}$ occurs when averaging over between 4 and 8 frequencies. Beyond 8 frequencies a point of diminishing returns is rapidly reached with regard to further reducing $\sqrt{V_x}$. Averaging over 4 frequencies, reduces the threshold offset (in dB), required to maintain a given false alarm rate by between 35% and 50%, depending upon clutter type, relative to that required for processing returns measured at a single frequency. At 8 frequencies, this reduction increases to between 40% and 65%. The results presented herein are substantiated by radar measurements of returns from a variety of different types of ground clutter that were made at Ku Band using range-azimuth resolution cell sizes of $36m^2$ and $108m^2$.

It should be pointed out that the effect of frequency-averaging on reducing the computed sample variance of the clutter, and lowering the resulting CFAR threshold level, is not the whole story with regard to the effect of frequency-averaging on improving fixed target detection performance. Fixed target detection performance is characterized by the probability of detection, P_d, realized at a specified false-alarm rate, P_{fa}, for a target whose return has a given value of target-to-clutter signal ratio T/C. Clearly, as stated earlier, the lowering of the CFAR threshold that results from frequency-averaging of the clutter returns improves fixed target detectability. However, for a fixed target aspect angle, the target returns are also randomly distributed as a function of frequency, with a standard deviation of about 5.6 dB. Therefore, the standard deviation of the frequency-averaged target returns also decreases as the number of

frequencies over which the returns are averaged is increased. This reduction in the standard deviation of the frequency-averaged target return provides an additional increase in detection probability, provided that the average value of T/C (for a fixed aspect angle) is greater than k_{σ_C}. Consequently, the improvement in fixed target detection performance due to the effect of frequency-averaging is somewhat greater than that due to the lowering of the CFAR threshold alone, as presented herein. It can be shown [6] that the maximum number of different frequencies over which returns can be averaged before the point of diminishing returns is reached with respect to improvement in fixed target detection performance is actually about 16 rather than the range of 4-8 indicated herein. However, a further treatment of this topic is beyond the scope of this report.

1.3 Organization of Report

The report is organized as follows. Chapter 2 describes the two-parameter CFAR target detection algorithm, using frequency-averaged clutter samples taken from a two-dimensional array of range-azimuth resolution cells. The term "two-parameter" refers to the fact that the CFAR threshold is computed as a function of both the sample mean and sample variance of the clutter.

Chapter 3 describes the evaluation of false-alarm probability, for the two-parameter CFAR target detection algorithm described in Chapter 2. The analysis is based upon the assumption that the frequency-averaged clutter samples processed by the CFAR algorithm of Chapter 2 are normally distributed. This assumption is a very good approximation to reality when

the clutter in the CFAR stencil is statistically homogeneous, with a log-normal amplitude distribution over space, and is logarithmically detected. Since the threshold is based upon the computed sample statistics, the resultant false alarm probability is governed by Student's "t" distribution, as described in Ref. [1].

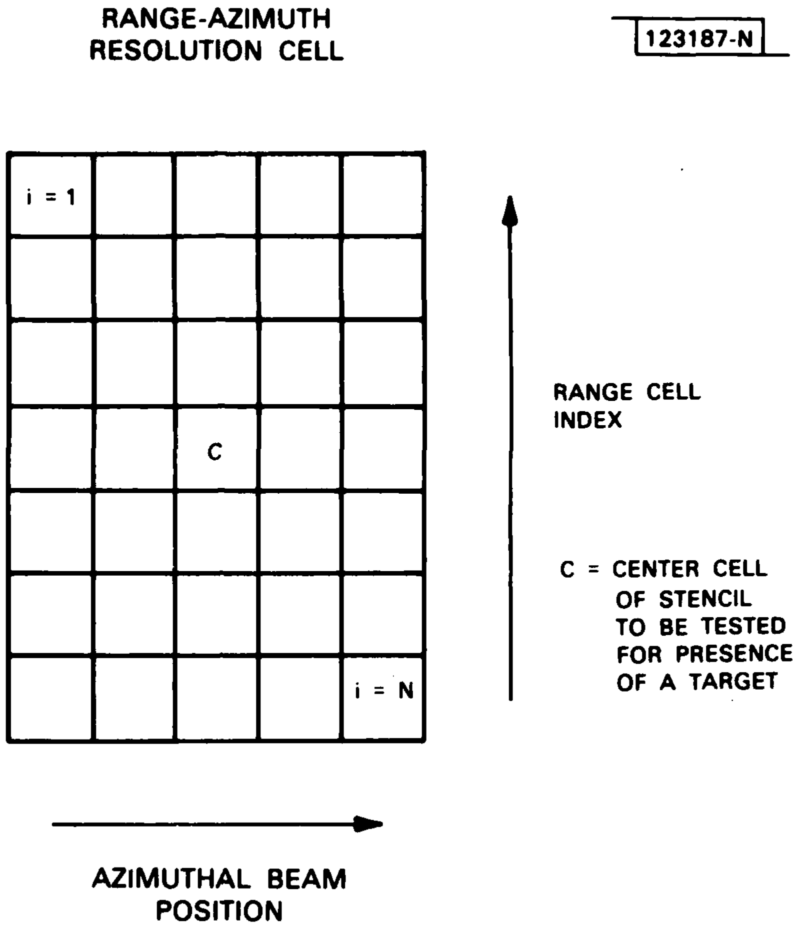
Chapter 4 describes the effect of frequency averaging on the clutter sample statistics X and V_x that were defined in Chapter 2. The principal effect of frequency averaging is to reduce the expected value of the sample variance, V_x. The expected value of the frequency-averaged clutter sample variance V_x is shown to consist of two components - the first is the variance of the intrinsic spatial distribution of the clutter amplitudes, σ_c^2, the second is due to the variability of the clutter returns from a given resolution cell as a function of frequency. It is this second component of V_x that is reduced by averaging the clutter samples from each resolution cell over a number of different frequencies.

In Chapter 5, experimentally measured clutter statistics for a variety of different types of ground clutter are presented. The experimentally observed reduction in clutter sample variance due to frequency-averaging is compared against the reduction predicted by the analysis of Chapter 4. Very good agreement between the experimentally measured and analytically predicted results is obtained.

2.0 THE TWO-PARAMETER CFAR ALGORITHM

Consider the two-dimensional array of range-azimuth resolution cells illustrated in Fig. 2.1. Let p be the size of this array in range resolution cells and q the azimuthal extent of the array in antenna beam positions. It is assumed that the cells in this array are non-overlapping with respect to both range and azimuth so that the returns from the individual cells in the array are all statistically independent. Normally p and q are odd numbers so that there will be a unique cell at the center of this array. All of the cells in this array, with the exception of the center cell, are assumed to contain local ground clutter returns. The size of the array is assumed to be small enough so that all the ground clutter within the array is homogenous - i.e., is from the same statistical population. Thus the effect of mixed types of ground clutter or clutter discontinuities such as tree lines are not considered here. The center cell is assumed either to contain clutter from the same population as the surrounding neighbors, or a target.

The presence of a target in the center cell of this array can be detected by comparing the amplitude of the return from that cell against a threshold level that is adaptively computed as a function of the sample mean and variance of the return amplitudes in remaining cells in the array, excluding the cell under test. If the return in the center cell exceeds this threshold, a target is declared to exist in that cell. The amplitudes of the clutter cells in the stencil surrounding the cell under test will generally be randomly distributed. The mean and variance of their



Let X_i be the frequency-averaged amplitude of the radar return in the ith cell of the stencil; let X_c be the frequency-averaged amplitude of the return in the center cell

Define: $\overline{X} \equiv \frac{1}{N} \sum_{i \neq C}^{N} X_i \qquad V \equiv \frac{1}{N} \sum_{i \neq C}^{N} (X_i - X)^2$

Detection Threshold $T \equiv X + k\sqrt{V}$

Then if $X_c > T$, a target is declared to be present in the center cell

Fig. 2.1 Diagram of a typical 2-dimensional CFAR stencil, illustrating the threshold detection process.

distribution will be a function of the type of clutter in those cells. Hence the threshold level must be set high enough so that the probability of a clutter return in the center cell exceeding this threshold and being declared a target will be maintained at a specified small value, usually on the order of 10^{-3} or less.

The amplitudes of the returns received from the clutter cells are randomly distributed with respect to both space and carrier frequency of the radar signal. The effect of the variability of the clutter amplitude returns as a function of frequency may be reduced by averaging the returns from a given resolution cell over several different frequencies prior to computing the sample mean and variance of the clutter in the stencil over space. The advantage of using the frequency-averaged returns from each resolution cell in the CFAR test, rather than the returns measured at only a single frequency, is that the threshold level required to maintain a given false alarm rate is lowered, thereby increasing the probability of target detection at any given value of false alarm rate. The probability of target detection depends solely on the target amplitude statistics and the threshold level, and is independent of the clutter level. Hence, the lower the threshold, the higher the probability of target detection.

Let $N=pq-1$ be the number of independent resolution cells in the two-dimensional stencil of Fig. 2.1, excluding the center cell. For convenience, let the individual cells in this stencil be identified by a one-dimensional index, $i=1,\ldots N$. Let the total number of independent frequencies at which the returns from each individual cell are measured by J, and the corresponding frequency index be $j=1,\ldots J$.

The returns from individual resolution cells in a given azimuthal beam cell can be measured nearly simultaneously over a number of different frequencies by transmitting J pulses at a fixed antenna beam position. For any reasonable number of frequencies, the total time required to collect these J pulse returns is usually only a few milliseconds. Hence even if the radar is moving relative to the ground, the motion of the radar during this interval is usually only a very small fraction of the resolution cell size, so that for all practical purposes the returns from all J pulses at a given beam position can be assumed to be from the same set of physical resolution cells on the ground. After each set of J pulse returns have been collected at a given beam position, the beam position is incremented by an amount approximately equal to the antenna beam width and the process repeated. The data comprising the CFAR stencil of Fig. 2.1 is then taken from the qJ pulse returns that were collected in covering q adjacent beam positions. Let the amplitude return from resolution cell i $(i=1,\ldots N=pq-1))$ collected at frequency j be designated as y_{ij}. Then the frequency-averaged returns from each resolution cell are formed as follows:

$$x_i = \frac{1}{J} \sum_{j=1}^{J} y_{ij} \tag{2.1}$$

The frequency average x_i then represents the best available estimate of the intrinsic amplitude level of the scatterers in resolution cell i of the stencil. Furthermore, designate i=0 as the index of the center cell of the stencil. Assume that the N cells in the stencil, outside of the central

cell, all contain local clutter of a given type. The sample mean and variance of the cell frequency averages, x_i, over the stencil may be then used as estimates of the mean and variance of the spatial probability distribution of the local clutter. These sample statistics are computed as follows. Clutter Sample Mean:

$$\overline{X} = \frac{1}{N} \sum_{i=1}^{N} x_i = \frac{1}{NJ} \sum_{i=1}^{N} \sum_{j=1}^{J} y_{ij} \tag{2.2}$$

Clutter Sample Variance:

$$V_x = \frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{X})^2 \tag{2.3}$$

It should be noted that the value of the sample mean, X, is the same regardless of whether this mean is taken of the frequency averages x_i, or of the raw frequency returns y_{ij} over both frequency and space. However, this is <u>not</u> true of the sample variance V_x. For example:

$$V_x \neq \frac{1}{NJ} \sum_{i=1}^{N} \sum_{j=1}^{N} (y_{ij} - \overline{X})^2$$

Note that the return x_o, from the center cell of the stencil is excluded from the above computation. To complete the description of the CFAR detection process, a threshold value is computed individually for each clutter stencil as follows:

$$T = \overline{X} + k\sqrt{V_x} \tag{2.4}$$

Where $\sqrt{V_x}$ is the square-root of equation (2.3). The frequency average of the center cell x_o is then compared against this threshold:

$$x_o > T?$$

If x_o is greater than T, a target is assumed to exist in the center cell. However, even if the center cell contains only clutter - i.e., if x_o is from the same statistical population as its neighbors x_i, $i=0$, there is still a finite probablity that $x_o > T$, resulting in a false alarm. The probability of such a false alarm occurring is a function of the value of T and the probability distribution of the x_i, as discussed in detail in the next section.

3.0 EVALUATION OF FALSE ALARM PROBABILITY

The variable used in the CFAR test is the frequency average of the radar returns, x_i. As is shown in Appendix B, if the x_i represent frequency averages of log-detector outputs in response to radar returns from log-normally distributed clutter, the distribution of the x_i will be approximately normal, with a mean and variance that are a function of the clutter statistics. This situation is illustrated in Fig. 3.1.

Then for any given threshold setting, T, the false alarm probability P_{fa} can be computed as the probability of $x > T$ and is represented by the area under the (approximately normal) probability density function f(x) of the frequency-averaged clutter amplitudes between T and infinity as illustrated in Fig. 3.1.

$$P_{fa}(T) = \int_{T}^{\infty} f(x)dx) \qquad (3.1)$$

Furthermore, if the density function f(x) is normal and the thresold T is computed as an offset from the mean, μ_x, of f(x) as:

$$T = \mu_x + k\sigma_x \qquad (3.2)$$

Then it can be easily shown that P_{fa}, as defined by equation 3.1, is merely a function of the threshold multiplier k, i.e.,

$$P_{fa}(k) = \int_{\mu_x+k\sigma_x}^{\infty} f(x)dx = \mathrm{Erfc}(k) \qquad (3.3)$$

In equation 3.2 above, the mean and variance of x, i.e., μ_x and σ_x^2 are assumed to be known parameters - i.e., constants. In reality,

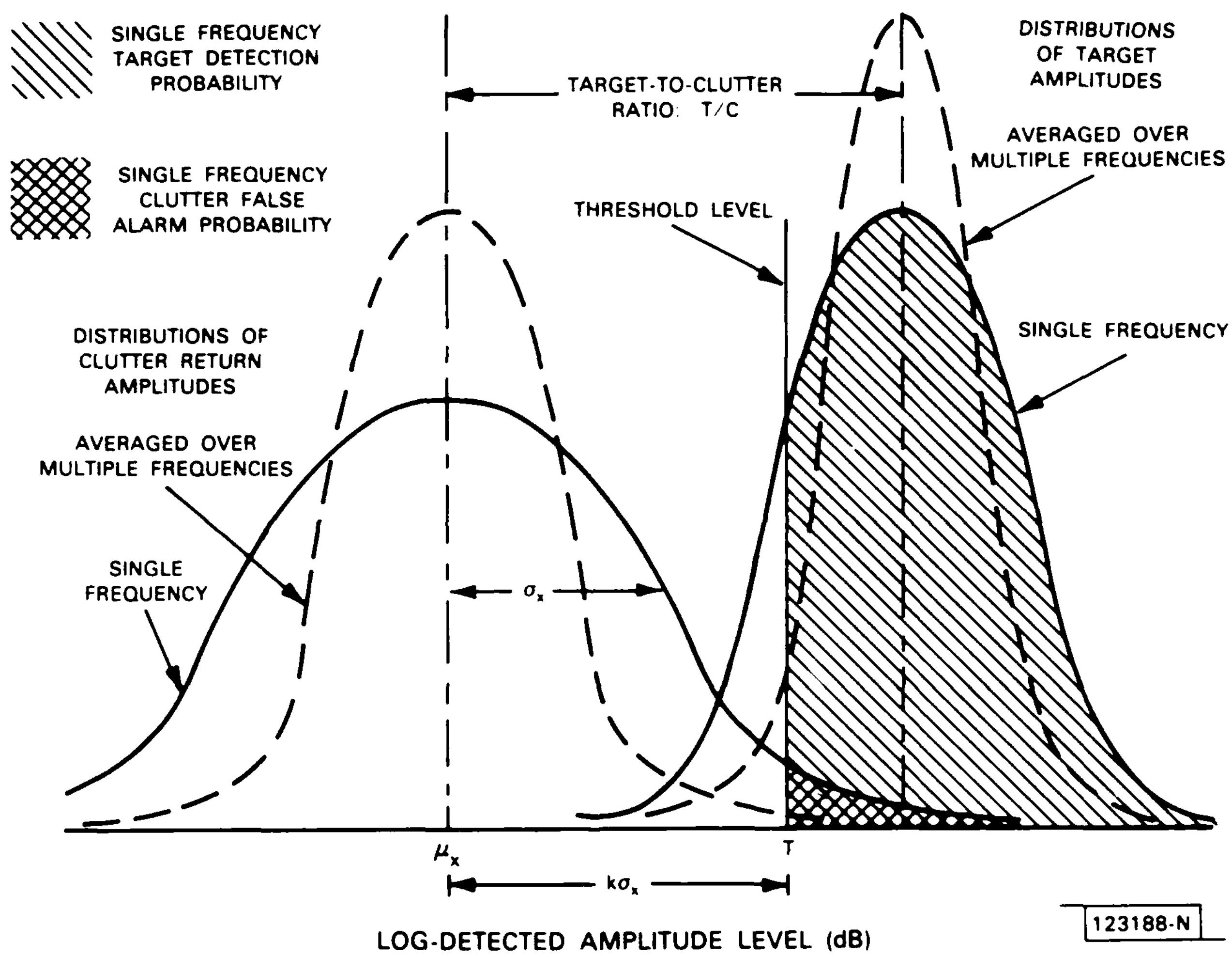


Fig. 3.1 Probability distributions of target and clutter returns for a given value of target-to-clutter ratio, indicating effect of detection threshold setting on target detection and clutter false alarm probabilities. Effect of frequency-averaging on narrowing of the distributions is also illustrated.

however, these parameters are unknown, and T can only be computed from estimates of μ_x and σ_x - i.e., from the sample statistics as in equation 2.4. As a result, T itself becomes a random variable. It can be shown [1] that when T is computed from the sample statistics X and V_x, (equation 2.4) and the x_i are normally distributed, the probability of false alarm is given by the following integral:

$$P_{fa}(N,k) = \int_{k\sqrt{\frac{N-1}{N+1}}}^{\infty} S_{N-1}(Z)dZ \tag{3.4}$$

where S_{N-1} is Student's distribution for N-1 degrees of freedom, and N is, as before, the number of resolution cells in the stencil used to compute the sample mean and variance, X and V_x. Specifically, Student's distribution $S_{N-1}(Z)$ is the distribution of the random variable:

$$Z = \sqrt{\frac{N-1}{N+1}} \left(\frac{x_o - x}{\sqrt{V_x}}\right)$$

where X and V_x are as defined by equations 2.2, 2.3 and the sample x_o is from the same population as the x_i, but is not included in the computation of X and V_x. As might be expected, as $N \to \infty$, Student's distribution approaches a zero-mean normal distribution of unit variance. Indeed for all practical purposes, when N >40 the value of P_{fa} computed from equation 3.4 approaches that of equation 3.3, for T defined by equation 2.4.

In summary, when the x_i are normally distributed, and T is computed as a function of the sample mean and variance in accordance with equation 2.4, the false alarm probability will be a unique function of the

threshold multiplier, k, and the number of resolution cells, N, over which the sample statistics are computed. However, in the sequel, the effect of N on P_{fa} will not be considered - i.e., N will be assumed to be fixed.

4.0 EFFECT OF FREQUENCY-AVERAGING ON THE CFAR THRESHOLD SETTING

The consequence of the previous argument that P_{fa} is only a function of the threshold multiplier, k, (the effect of N being neglected since N is usually fixed) is that in order to maintain a specified P_{fa}, the absolute value of the threshold setting T will be proportional to both the sample mean, X, and the square root of the sample variance, $\sqrt{V_x}$, in accordance with equation 2.4. However in order to maximize the probability of target detection P_d, the threshold should be kept as low as possible, consistent with maintaining the specified value of P_{fa}. The question at hand then is how can frequency averaging of the data be used to lower the value of the threshold setting, T, without raising P_{fa}?

The major benefit provided by frequency averaging is that, for most types of clutter, frequency averaging significantly reduces the expected value of the clutter sample variance, V_x. This allows the CFAR threshold level required to maintain a given false alarm rate to be decreased correspondingly, as per equation 2.4, with the result that higher detection probabilities can be realized for a given value of clutter false alarm rate.

It is shown in Appendix A that the expected value of the sample variance, V_x, of the cell frequency averages, X_i, is given by the following expression.

$$E[V_x] = \frac{N-1}{N}\left(\frac{1}{J}\ E[\sigma_{y/m_x}^{2}] + \sigma_c^{2}\right) \qquad (4.1)$$

In the above expression σ_c^2 is the variance of the spatial distribution of the intrinsic clutter amplitudes, and E $[\sigma_{y/m_x}^{2}]$ is the expected value of the variance of the clutter amplitude returns received from a resolution cell containing clutter of intrinsic amplitude level m_x, and is a measure of the variability of the returns from that cell over frequency. J is the number of frequencies averaged, and N is the number of independent resolution cells in the stencil. N should be kept small enough so that the stencil contains only homogenous clutter of variance σ_c^2, but large enough so that the sample statistics, X and V_x, are good estimates of the true mean and variance of the local clutter. A reasonable compromise is to choose a stencil having between 20 and 40 resolution cells. The value of σ_c^2 is difficult to measure and will never be known exactly. As previously discussed, it is a characteristic of the local clutter, and may vary greatly within any given radar scene. However, as can be seen from equation 4.1, σ_c^2 can be estimated from V_x when J is large - i.e., when the clutter returns from a given ground patch are averaged over a large number of frequencies.

4.1 Effect of Logarithmic Detection

If the clutter returns are logarithmically detected, the output of the log detector, being proportional to the logarithm of the signal voltage, will be linear in terms of dB relative to some arbitrary reference. Hence, the averages, x_i, of the log detector outputs can also be expressed in dB relative to this reference. Clearly, the choice of the reference level only effects the origin of the dB scale. Hence, although the mean value, $\overline{X}$, of the x_i will be dependent upon this choice of origin, the variance, V_x, of the x_i will not. Therefore the terms, σ_c^2 and $E[\sigma^2_{y/m_x}]$, in equation 4.1 above can be expressed in dB^2, and are independent of the choice of origin for the dB scale.

Furthermore, it is shown in Appendix C that for the case of a logarithmic detector, the quantity $\sigma^2_{y/m_x} = 31\ dB^2$ is a constant, independent of m_x, whose value is:

$$\sigma^2_{y/m_x} = 31\ dB^2.$$

4.2 Clutter Sample Variance as a Function of Frequencies Averaged and Intrinsic Clutter Variance

Substituting this into equation 4.1 gives the following result for the expected value of the stencil variance:

$$E[V_x] = \frac{N-1}{N}\left(\frac{31}{J} + \sigma_c^2\right),\ dB^2 \qquad (4.2)$$

The effect of frequency averaging on variance reduction is now clear. Frequency averaging only reduces that component of the total observed clutter

variance - i.e., 31 dB^2 - that is attributable to variation of the amplitude of the clutter returns with frequency. Hence the effectiveness of frequency averaging depends upon the magnitude of the intrinsic clutter variance, σ_c^2, relative to 31 dB^2, or equivalently the standard deviation of the clutter relative to 5.6 dB. Clearly the smaller the intrinsic standard deviation of the clutter, σ_c, the more effective frequency averaging will be in reducing the expected value of the stencil variance, V_x. This phenomenon will be explored in more detail and compared with a variety of experimental clutter measurements in the next section.

4.3 Effect of Frequency-Averaging on Threshold Setting

As mentioned earlier, the effect of frequency averaging on CFAR detection performance is to lower the threshold setting required to maintain a given false alarm rate, thereby improving detection performance by increasing the probability of target detection without compromising false alarm performance. This can be seen by substituting equation 4.2 for the expected value of V_x into equation 2.4 for the threshold setting, and is also illustrated in Fig. 4.1. Hence an expression for the effect of frequency averaging on the expected value of the threshold setting required to maintain a given value of false alarm rate is given by:

$$E[T] = \mu_x + k\sqrt{\frac{N-1}{N}\left(\frac{31}{J} + \sigma_c^2\right)} \quad \text{dB} \qquad (4.3)$$

Or equivalently, the expected value of the threshold setting relative to the mean value of the clutter is:

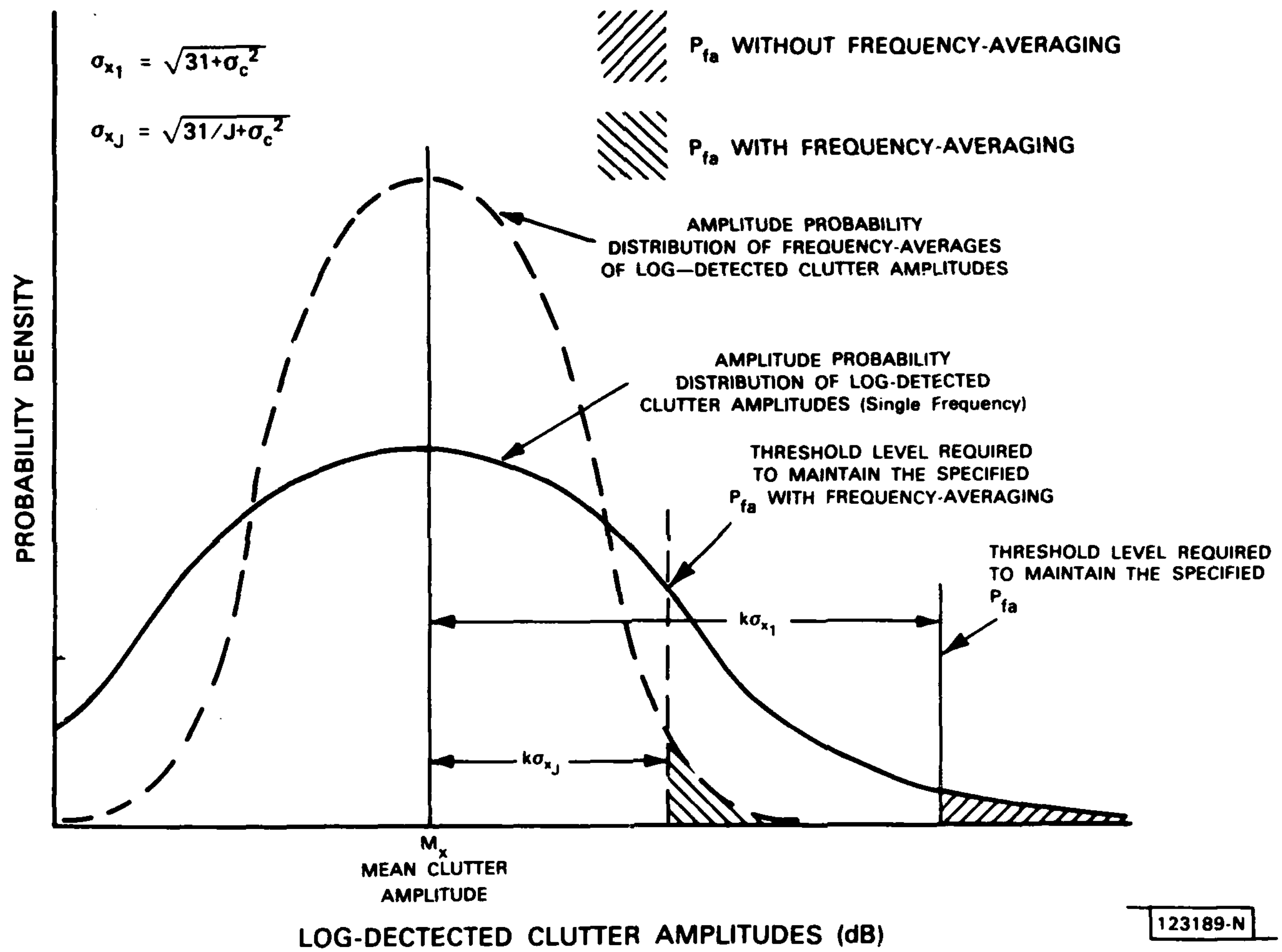


Fig. 4.1 Probability distribution of log-detected clutter amplitudes with and without frequency-averaging illustrating the effect of frequency-averaging on narrowing the distribution and reducing the threshold level required to maintain a specified P_{fa}.

$$E[T-\mu_x] = k\sqrt{\frac{N-1}{N}\left(\frac{31}{J} + \sigma_c^2\right)} \quad \text{dB} \tag{4.3a}$$

As previously discussed, the false alarm rate corresponding to this threshold setting is determined by the value of the threshold multiplier k, for fixed N. Note that equation (4.3a) for the relative threshold setting is independent of the dB scale origin chosen for the log detector outputs.

5.0 EXPERIMENTAL VERIFICATION

In the preceding section, a quantitative relation, equation (4.2), was obtained for the expected value of the clutter sample variance as a function of the number of frequencies and number of resolution cells averaged, and the variance of the underlying spatial amplitude distribution of the clutter in the stencil. Consequently, equation 4.2 may be used to predict the effect of frequency averaging on the sample variance computed for different types of clutter.

For this purpose, each clutter type may be uniquely characterized by the variance, σ_c^2, of its intrinsic spatial amplitude distribution. For example, smooth clutter such as mown, level fields, or pavement is characterized by a spatial amplitude distribution having a very low value of σ_c^2, whereas tree cover or rough terrain would be characterized by a spatial amplitude distribution having a much higher value of σ_c^2. Ideally, the values of σ_c^2 for each clutter type can be obtained as the limit of the sample variance, over space, of clutter returns that have been frequency-averaged over an infinite number of <u>independent</u> frequencies. Clutter returns obtained at two different frequencies may be shown to be statistically independent if the difference between the two frequencies is greater than the radar bandwidth [7]. Obtaining measurements at an infinite number of frequencies is clearly impractical; however in most cases of practical interest, the limiting variance of the spatial clutter amplitude distribution will be approached asymptotically, within the margin of experimental error, when clutter returns are averaged over more than 32 frequencies.

5.1 The Variance Reduction Factor

Of particular interest is the reduction in the expected value of clutter sample variance, V_x, that results from averaging the clutter returns over a specified number of frequencies. It is the ratio of the expected value of V_x, when the clutter samples are averaged over J frequencies, to the expected value of V_x computed from clutter samples measured at only a single frequency (i.e., no frequency averaging). This variance reduction ratio is defined as $\rho(J, \sigma_c)$ and is obtained from equation 4.2 as follows:

$$\rho(J,\sigma_c) = \frac{E[V_x(J)]}{E[V_x(1)]} = \frac{31 + J\sigma_c^2}{J(31+\sigma_c^2)} \quad \text{where } \sigma_c^2 \text{ and 31 are units of } dB^2. \tag{5.1}$$

The functional dependence of this variance reduction factor on both the number of frequencies, J, averaged, and upon the standard deviation, σ_c, of the intrinsic spatial clutter amplitude distribution is specifically indicated.

5.2 Description of Experimental Clutter Measurement Data

Experimentally derived clutter statistics for a wide variety of commonly encountered types of ground clutter have been measured at Ku band and reported in [2]. The results of these measurements are summarized in Tables 5.1 and 5.2. Table 5.1 is a summary of clutter measurements made at Stockbridge, NY in the summer of 1977, using a $100m^2$ range-azimuth resolution cell. The measurements were averaged over 32 different frequencies spanning a bandwidth of 500 MHz. Table 5.2 summarizes the results of a

TABLE 5.1. Summary of Clutter Parameters*
Stockbridge, NY - Late Summer, 1977

Clutter Type	(1) $E\{\gamma(dB)\}$	(2) $\sqrt{D^2\{\gamma(dB)\}}$	(3) $E\{\overline{\gamma(dB)}\}$	(4) $\sqrt{D^2\{\overline{\gamma(dB)}\}}$	(5) $E\{\gamma\}(dB)$	(6) $D^2\{\gamma(dB)\}/D^2\{\overline{\gamma(dB)}\}$
Deciduous Trees	-11.6 dB	4.9 dB	-11.6 dB	1.6 dB	-8.8 dB	10.1
Coniferous Trees	-14.1	5.0	-14.1	2.3	-11.2	4.8
Scrub	-12.5	5.0	-12.5	1.4	-9.6	12.5
Grass	-13.3	5.0	-13.3	1.2	-10.5	18.2
Dirt	-13.7	4.9	-13.7	1.0	-11.0	24.0

(1) Mean value of $10\{\log_{10}\gamma\}$

(2) Standard deviation of $\{10 \log_{10}\gamma\}$

(3) Mean value of the 32-pulse frequency average of $\{10 \log_{10}\gamma\}$

(4) Standard deviation of the 32-pulse frequency average of $\{10 \log_{10}\gamma\}$

(5) Mean reflectivity converted to dB, i.e., $10 \log_{10}(E\{\gamma(m^2/m^2)\})$ computed as
$(5) = (1) + 0.1152*(2)^2$

(6) Variance reduction attributable to frequency averaging $\triangleq [(2)/(4)]^2$

*Resolution cell (9 m, range) x (12 m, cross-range)

TABLE 5.2. Summary of Clutter Parameters*
Ft. Devens, MA - Summer, 1978

Clutter Type	(1) $E\{\gamma(dB)\}$	(2) $\sqrt{D^2\{\gamma(dB)\}}$	(3) $E\{\overline{\gamma(dB)}\}$	(4) $\sqrt{D^2\{\overline{\gamma(dB)}\}}$	(5) $E\{\gamma\}(dB)$	(6) $D^2\{\gamma(dB)\}/D^2\{\overline{\gamma(dB)}\}$
Meadow/Scrub	-12.7 dB	5.2 dB	-12.7 dB	1.8 dB	-9.6 dB	8.4
Deciduous Trees**	-9.2	5.6	-9.2	2.7	-5.6	4.3

*Resolution cells approximately 6 m by 6 m
Definitions for tabulated parameters given with Table I except that the frequency average appearing in (3), (4), and (6) used 16 pulses instead of 32.

**Deciduous trees were not densely and uniformly spaced as they were in the Stockbridge tests; in addition, they were on the facing edge of a small hill. The 11° slope of the hillside introduces ~2 dB so that the mean return reported should be scaled down by 2 dB for an equivalent flat earth measurement.

similar set of clutter measurements made at Fort Devens, MA in the summer of 1978 using a $40m^2$ range-azimuth resolution cell and 16 different frequencies spanning the same 500 MHz bandwidth.

The smaller range-resolution used at Fort Devens corresponds to a wider radar signal bandwidth. Therefore, in order to maintain statistical independence between the measurements made at different frequencies at the wider signal bandwidth, the differential between adjacent frequencies had to be increased. Since the overall frequency agility bandwidth of the radar was limited to 500 MHz, the required increase in frequency differential was realized by reducing the number of frequencies spanning that bandwidth from 32 to 16.

The clutter statistics are represented in terms of the clutter reflectivity, γ, which is a dimensionless quantity corresponding to apparent cross-section of the clutter return in m^2, divided by the projection of the ground area covered by the radar's range-azimuth resolution cell onto a plane normal to the radar line-of-sight. Another dimensionless measure of clutter reflectivity that is often used is σ_o, defined as:

$$\sigma_o = \frac{\text{apparent radar cross-section}}{(\text{ground area of radar resolution cell})}$$

The two parameters are related as follows:

$$\gamma = \sigma_o \csc \theta_d$$

where θ_d is the depression angle of the radar-line-of-sight with respect to the ground plane. Because of this additional normalization, γ is an intrinsic clutter parameter that is more nearly independent of depression angle over a wide range of values of θ_d [4].

As pointed out in the last chapter, since the clutter amplitude returns are logarithmically detected, they may be represented directly in terms of dB. Thus the above normalization only results in a shift of the dB scale origin and has no effect on the standard deviation (or variance) of the clutter returns, when expressed in dB. Therefore the previously derived expressions for clutter sample variance in dB are still applicable when the clutter returns are expressed in terms of γ (dB).

The meaning of the various columns of Tables 5.1 and 5.2 are defined in Table 1. However a few amplifying comments may be helpful. Column 1 is the mean value of the log-detected clutter returns in dB averaged over both space and frequency. It is computed using the second expression for $\overline{X}$ in equation 2.2. Column 3 is the spatial mean, over the stencil, of the frequency averages of the log-detected clutter returns for each cell of the stencil. It is computed using the definition of the frequency average given by equation 2.1 and the first expression in equation 2.2. As would be expected from the equality of the two expressions in equation 2.2, the values in these two columns are identical. Column 2 represents the sample standard deviation of the clutter returns measured at only a single frequency, in effect $\sqrt{V_X}(1)$. Column 4 represents the sample standard deviation of the frequency-averages as defined by equation 2.3, in effect $\sqrt{V_X}(J)$. Column 5 again represents the mean value of the clutter, but where the averaging is done <u>prior</u> to logging. Hence, the difference between columns (1) and (3) on the one hand and column (5) on the other, is that the values in column (1) and (3) are computed as the averages of the log-detected clutter

signal voltages, whereas column (5) is computed as the logarithm of the average clutter signal voltage. It can be shown that the two quantities are related by the expression given in footnote (5) of Table 5.1. In that expression the numbers in parentheses, including the (2), refer to the column numbers of the table.

Finally, column (6) of the tables is the reduction in the sample variance of the frequency averaged clutter returns that results from frequency averaging, and is computed as the square of the ratio of column (2) (the sample standard deviation for returns measured at a single frequency) to column (4) (the sample standard deviation for returns frequency-averaged over J frequencies, where J=32 for Table 5.1 and J=16 for Table 5.2). Thus the data in column (6) of the tables corresponds to the reciprocal of the variance reduction factor $\rho(J,\sigma_c)$ defined by equation (5.1).

5.3 Estimation of Intrinsic Spatial Clutter Standard Deviation From Measured Clutter Sample Statistics

In order to compare the experimentally measured variance reduction factors against the predicted values from equation 5.1, it is first necessary to determine the value of the standard deviation of the intrinsic spatial distribution of clutter amplitudes, σ_c, (corresponding to averaging over an infinite number of frequencies) for each of the clutter types listed in Tables 5.1 and 5.2.

This process is a bit tricky because only two frequency-averaged data points are available for each type of clutter, i.e., single-frequency (no averaging) and the averages taken over either 16 or 32 frequencies, and also because of probable random measurement and calibration errors in the data, as will be discussed further below. Ideally, an estimate of σ_c^2

would be obtained for each clutter type by computing the experimentally derived variance reduction factor $V_x(J)/V_x(1)$ from the data measured at k different frequencies, at a number m (>3) different values of J, $J_m \leq k$, so as to obtain a sequence of values of $V_x(J_m)/V_x(1)$ for that type of clutter, for m different numbers of frequencies averaged. A least squares fit of equation 5.1 would then be made to the resulting sequence with $\sigma_c{}^2$ as a parameter. The value of $\sigma_c{}^2$ that produced the best fit of equation (5.1) to this sequence of m values of $V_x(J_m)/V_x(1)$ would be the best estimate of $\sigma_c{}^2$ for that type of clutter. Unfortunately, the values of $V_x(J)$ cited herein are only available at two values of J for each clutter type: either J=16 or 32, and trivially for J=1. Although a fit of equation (5.1) could be obtained that would pass through the resulting two data points, i.e., $V_x(1)/V_x(1)=1$ and $V_x(J)/V_x(1)$, for each clutter type, the problem is further complicated by the fact that there is also an apparent calibration error in the data, as discussed below.

From equation (4.2) it follows that:

$$\sigma_c{}^2 = \frac{N}{N-1} \; E\left[V_x(J)\right] - \frac{31}{J} \tag{5.2}$$

The number of resolution cells used to obtain the values of $V_x(J)$ tabulated in Tables 5.1 and 5.2 ranged from 200-800. Hence for all practical purposes the factor $(N/N-1) \approx 1$. However a calibration discrepancy is indicated by the fact that with one exception, the values of $\sqrt{V_x}(1)$ listed in column (2) of Tables 5.1 and 5.2 are all less than $\sqrt{31}$, so that if the corresponding values of $V_x(1)$ were substituted into

equation 5.2, negative values of σ_c^2 would be obtained - a clearly unrealistic result. Consequently the values of $V_x(1)$ indicated in Tables 5.1 and 5.2 appear to be somewhat low. The problem appears to be due to errors in the calibration of the logarithmic detector. Although the exact cause of this discrepancy is unknown, a likely cause appears to be the bandwidth limit of the logarithmic detector. The log-detector is effectively calibrated at "D.C." - i.e., the detector outputs are measured at constant input signal levels that are incremented in a step-by-step manner in order to obtain the detector calibration curve. However, if the amplitude variation of the clutter returns between adjacent range resolution cells, divided by the range resolution in units of time, is greater than the detector bandwidth, the magnitude of the cell-cell fluctuations in clutter amplitude will be attenuated. Consequently, the variance of the log-detected returns will be somewhat less than their true variance, giving rise to the observed behavior. As a result, in order to use equation (5.2) to estimate σ_c^2 for the data in Tables 5.1 and 5.2, the constant: 31 dB^2 will be replaced by the quantity $V_x(1)-\sigma_c^2$, where $V_x(1)$ is obtained from column 2 of Tables 5.1 and 5.2. This will force equality of equation (5.2) when J=1, regardless of the measured value of $V_x(1)$. Note that the value of $V_x(1)$ is only weakly influenced by the value of the clutter variance σ_c^2, in those cases where $\sigma_c < 3$ dB. Hence assuming that $N/N-1 \approx 1$, a modified form of equation (5.2), adjusted for the effects of the apparent calibration error is:

$$\hat{\sigma}_c^2 = V_x(J) - \frac{V_x(1)-\hat{\sigma}_c^2}{J}$$

Or solving the $\hat{\sigma}_c^2$ gives:

$$\hat{\sigma}_c^2 = \frac{J}{J-1} V_x(J) - \frac{V_x(1)}{J-1} \qquad (5.3)$$

The number of frequencies, J, is either 32 or 16, depending upon whether the data from Table 5.1 or 5.2 is being adjusted. The expected clutter variance at the indicated number of frequencies, $E[V_x(J)]$, is obtained as the square of the standard deviation of the frequency averages given in column (4) of the tables. Estimated values of $\hat{\sigma}_c$ for each clutter type in Tables 5.1 and 5.2 are obtained by substituting the observed values of $V_x(1)$ and $V_x(J)$ into equation (5.3). The results are tabulated in Table 5.3 together with the number of frequencies and the experimentally measured variance reduction factor (the reciprocal of column (6) in Tables 5.1 and 5.2). Having estimated the value of the intrinsic clutter variance, σ_c, for each clutter type listed in Tables 1 and 2, one can now compare the corresponding experimentally derived variance reduction factors, $\rho(J,\hat{\sigma}_c)$, against the values predicted using equation 5.1.

5.4 Comparison of Experimental Data with Theoretical Predictions

It is somewhat more useful to portray the effect of frequency-averaging in terms of the reduction in standard deviation of the clutter rather than the reduction in variance, since it is the estimated standard deviation that affects the detection threshold level. Therefore the

frequency-average standard deviation reduction factor $\sqrt{\rho(J, \sigma_c)}$, based upon equation 5.1, is plotted in Fig. 5.1 as a function of the number of frequencies averaged, J, for clutter standard deviations, σ_c, ranging from 0 to 3.0 dB, in steps of 1.0 dB. Note that the curve for σ_c = 0 dB represents the ideal reduction of $\sqrt{\rho(J,0)} = \sqrt{1/J}$.

Also plotted in Fig. 5.1, for comparison, are the experimentally derived standard deviation reduction factors based upon the measurement data tabulated in Tables 5.1 and 5.2. The estimated value of $\hat{\sigma}_c$ for each experimental data point, that was obtained using equation (5.3) and is tabulated in Table 5.3, is also indicated in Fig. 5.1. The agreement between the experimentally derived data points and the theoretical reduction curves predicted using equation (5.1) appears to be quite good for all cases considered, i.e., the values of $\hat{\sigma}_c$ estimated for each experimentally derived data point are within less than 0.5 dB of the theoretical value of σ_c that would yield the observed value of reduction at the indicated number of frequencies.

As mentioned previously, the experimentally derived data tabulated in Tables 5.1 and 5.2 appear to be subject to a number of measurement and calibration errors. Such errors can effect the estimated values of σ_c, tabulated in Table 5.3, by as much as 0.5 dB. The effect of these errors on $\rho(J, \sigma_c)$ is believed to be less because this parameter is computed as the ratio of two measurements, and systematic errors in the measurements would tend to cancel in this ratio. Two sources of such errors are known.

TABLE 5.3

Clutter Type	No. Frequencies	$\sqrt{V_x}(1)$ (1)	$\sqrt{V_x}(J)$ (2)	σ_c (3)	$\rho(J,\sigma_c)$ (4)	$\sqrt{\rho}(J,\sigma_c)$
Stockbridge, NY:						
Deciduous Trees	32	4.9	1.6	1.37	.107	0.326
Coniferous Trees	32	5.0	2.3	2.16	.212	0.46
Scrub	32	5.0	1.4	1.10	.078	0.28
Grass	32	5.0	1.2	0.82	.058	0.24
Dirt	32	4.9	1.0	0.51	.042	0.20
Ft. Devens, MA:						
Meadow/Scrub	16	5.2	1.8	1.28	0.120	0.346
Deciduous Trees	16	5.6	2.7	2.38	0.232	0.482

(1) From Column (2) of Tables 1 and 2.
(2) From Column (4) of Tables 1 and 2.
(3) Estimated using equation (5.2) of text.
(4) Reciprocal of Column (6) of Tables 1 and 2.

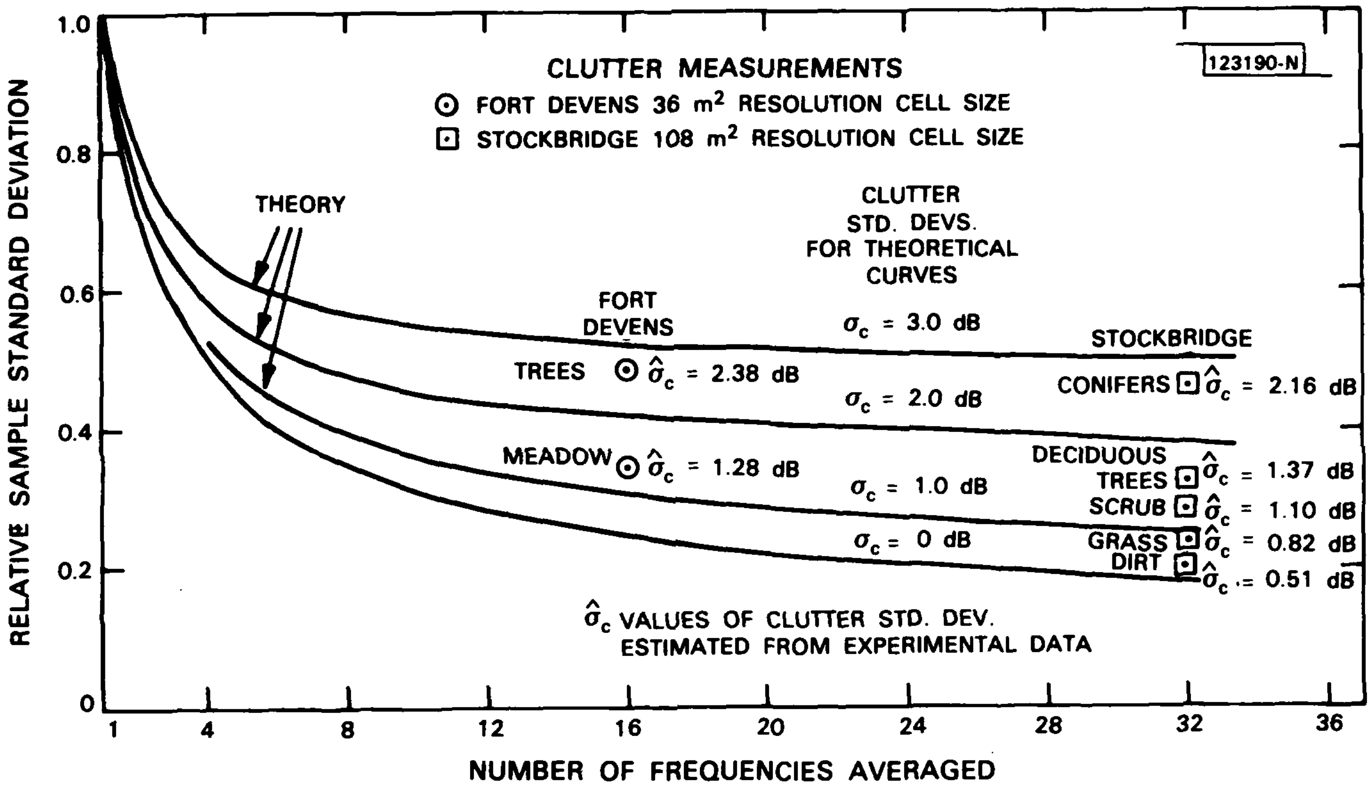


Fig. 5.1 Effect of frequency-averaging in reducing clutter sample standard deviation (square-root of sample variance of frequency-averages over space) for different clutter types.

First, as already mentioned, there appears to be a discrepancy of about 0.5 dB in the single-frequency clutter standard deviations tabulated in columns (2) of Tables 1 and 2. Although this discrepancy is small and appears to be on the order of the expected experimental error, it appears to be too systematic to be random. The exact cause is unknown, but as mentioned previously, it is believed to be due to dynamic miscalibration of the logarithmic detector at high signal bandwidths.

Second, there is an effective quantization error of 0.1 dB in the data of Tables 1 and 2, in that the data were only tabulated with that precision. However an error of 0.1 dB in the value of $\sqrt{E[V_x]}$ listed in column (4) of the tables can result in an error in the estimated value of σ_c obtained from equation 5.2 of as much as 0.3 dB, at the lower values of σ_c.

ACKNOWLEDGMENTS

Most of this report is based upon work that was originally performed at Lincoln Laboratory by Dr. Robert E. Stovall, but that was unfortunately never published. The author is especially indebted to Dr. Stovall for making his unpublished work available, for many helpful discussions that clarified a number of subtle issues, and for a review of the material presented herein. I would also like to thank Richard L. Harter for a number of clarifying comments and Dr. Leslie Novak for the derivation in Appendix C.

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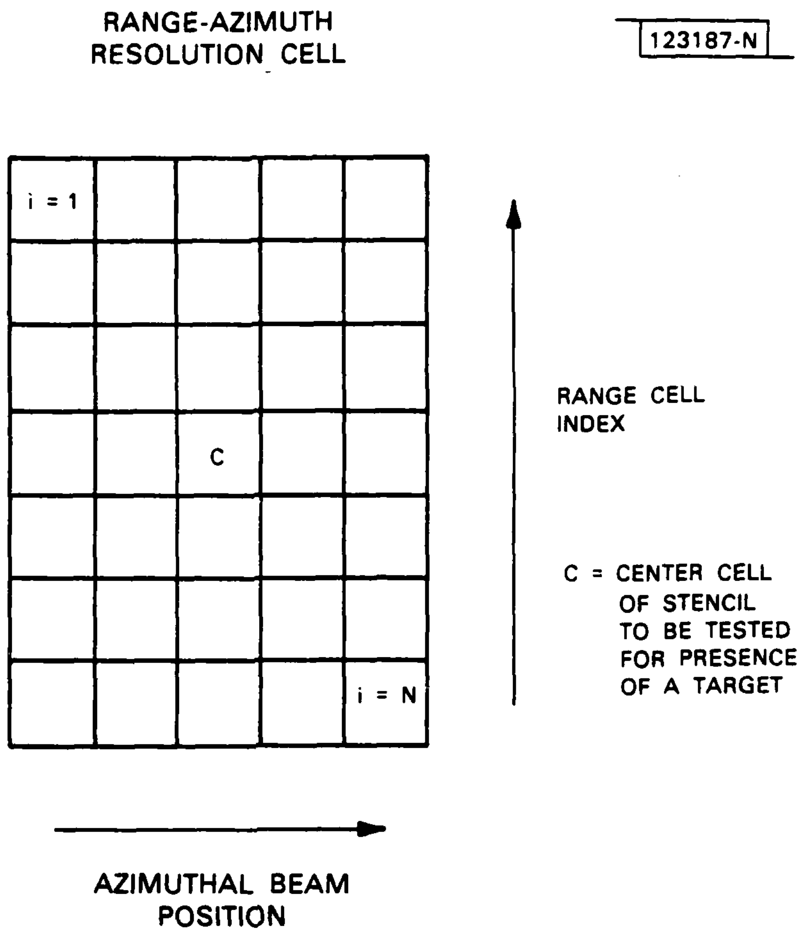
APPENDIX A

Derivation of CFAR Stencil Variance as a Function of Stencil Size, Number of Frequencies Averaged, and Clutter Statistics

Consider the two-dimensional range-azimuth CFAR stencil shown in Fig. A-1, consisting of N=pxq independent resolution cells, where p is the extent of the stencil in range cells, and q is the azimuthal extent in beam positions. Let each of the N resolution cells in this stencil be identified by an index i, i=1, ... N. Let returns be collected from each cell in the stencil at J different frequencies, sufficiently separated so that the corresponding J returns from the given cell are all statistically independent. Let the amplitude of the return from cell i for frequency j be designated y_{ij}. The distribution of the y_{ij} is two-dimensional. One dimension represents frequency, the other the spatial variation (i.e., cell location) in the stencil. The mean value of the return from each cell, over frequency, represents the intrinsic amplitude of the clutter in that cell, and is a random variable with respect to space. Let the mean value of the frequency returns from cell i be z_i. Then the two-dimensional probability distribution function of the returns can be represented by:

$$p(y,z) = p_{y/z}(y/z)\ p_z(z) \tag{A-1}$$

Note that y and z are <u>not</u> independent rv.; in effect z is the mean value of the distribution $p_{y/z}(y/z)$.



Let X_i be the frequency-averaged amplitude of the radar return in the ith cell of the stencil; let X_c be the frequency-averaged amplitude of the return in the center cell

Define: $\overline{X} \equiv \frac{1}{N} \sum_{i \neq C}^{N} X_i \qquad V \equiv \frac{1}{N} \sum_{i \neq C}^{N} (X_i - X)^2$

Detection Threshold $T \equiv X + k\sqrt{V}$

Then if $X_c > T$, a target is declared to be present in the center cell

Fig. A-1. Diagram of a typical 2-dimensional CFAR stencil, illustrating the threshold detection process.

$$z = \int_0^\infty y p_{y/z}(y/z) dy \qquad (A.2)$$

However as argued by Stovall[2], $P_{y/z}(y/z)$ and $P_z(z)$ are two distinct identifiable distributions, $P_z(z)$ is the spatial distribution of the mean clutter amplitude and is generally log-normal, whereas $P_{y/z}(y/z)$ is the amplitude distribution of the frequency returns from a given cell and is generally exponential or Rayleigh depending upon whether the amplitude y is represented in terms of power or voltage.

The purpose of this appendix is to derive an expression for the expected value of the sample variance of the frequency-averages of the clutter returns from the N resolution cells contained in the range-azimuth stencil illustrated in Fig. A.1, as a function of the number of frequencies averaged, stencil size, and the statistics of the intrinsic spatial probability distribution of clutter amplitude.

Define the following terms:

The sample average, x_i, over frequency, of the returns from cell i:

$$x_i = \frac{1}{J} \sum_{j=1}^{J} y_{ij} \qquad (A.3)$$

The sample average, $\overline{X}$, of returns over the entire stencil:

$$\overline{X} = \frac{1}{NJ} \sum_{i=1}^{N} \sum_{j=1}^{J} y_{ij} = \frac{1}{N} \sum_{i=1}^{N} x_i \tag{A.4}$$

The sample variance, V, over the stencil of the frequency-averaged returns from the individual cells:

$$V = \frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{X})^2 \tag{A.5}$$

Furthermore, using (A.2) it follows that for a given cell i:

$$E_{z_i}[y_{ij}] = z_i \tag{A.6a}$$

Therefore, it also follows from (A.3) that:

$$E_{z_i}[x_i] = z_i \tag{A.6b}$$

where the notation E_{z_i} signifies conditional expectation given z_i.

Also define:

$$E_{z_i}[(y_{ij}-z_i)^2] = \sigma^2_{y/z_i} \qquad (A.7)$$

(A.7) represents the variance of the frequency-diverse returns from a given resolution cell, the returns from which have a mean value z_i. It should be noted that, in general, σ^2_{y/z_i} may be a function of z_i and is therefore not necessarily constant.

Finally the statistics of the underlying spatial clutter distribution are defined by:

$$\mu_z = E[z_i] \qquad (A.8a)$$

$$\sigma_z^2 = E[(z_i-\mu_z)^2] \qquad (A.8b)$$

Given the preceding definitions, we will now derive an expression for the expected value of the sample variance of the frequency averages E[V], equation (A.5), in terms of the number of cells, N, the number of frequencies, J, the variance of the frequency-diverse returns, σ^2_{y/z_i}, and the spatial clutter amplitude statistics, μ_z and σ_z^2.

Taking the expected value of equation (A.5), one can write:

$$E[V] = E\left[E_{z_i}\left(\frac{1}{N}\sum_{i=1}^{N}(x_i - X)^2\right)\right]$$

$$= \frac{1}{N}\sum_{i=1}^{N} E\left[E_{z_i}\left((x_i - X)^2\right)\right] \qquad (A.9)$$

Expand the inner term of the above summation as follows:

$$E_{z_i}\left[(x_i - \bar{X})^2\right] = E_{z_i}\left[(x_i - z_i) - (\bar{X} - z_i)^{\,2}\right]$$

$$= E_{z_i}\left[(x_i - z_i)^2 - 2\,(x_i - z_i)(\bar{X} - z_i) + (\bar{X} - z_i)^2\right] \qquad \text{(A.10)}$$

Now expand the expectations of each of the three terms in (A.10) individually as follows:

For the first term, using the definition of x_i from equation A.3:

$$E_{z_i}\left[(x_i - z_i)^2\right] = \frac{1}{J^2} E_{z_i}\left[\sum_{j=1}^{J} (y_{ij} - z_i)\right]^2$$

Expanding the square of the summation and taking the expectation inside the resulting double summation gives:

$$E_{z_i}\left[(x_i - z_i)^2\right] = \frac{1}{J^2} \sum_{j=1}^{J} \sum_{k=1}^{J} E_{z_i}\left[(y_{ij} - z_i)(y_{ik} - z_i)\right]$$

Since the frequency samples are assumed to be independent it follows that:

$$E_{z_i}\left[(y_{ij} - z_i)(y_{ik} - z_i)\right] = E_{z_i}(y_{ij} - z_i)\, E_{z_i}(y_{ik} - z_i) = 0 \text{ for } j=k$$

$$E_{z_i}\left[(y_{ij} - z_i)(y_{ik} - z_i)\right] = E_{z_i}\left[(y_{ij} - z_i)^2\right] \qquad \text{for } j=k$$

$$= \sigma^2_{y/z_i}$$

Since j=k exactly J times in the above double summation, it follows that:

$$E_{z_i}(x_i - z_i)^2 = \frac{1}{J^2} J \, \sigma^2_{y/z_i} = \frac{1}{J} \, \sigma^2_{y/z_i} \tag{A.11}$$

For the second term in (A.10):

$$-2E_{z_i}\left[(x_i - z_i)(\bar{X} - z_i)\right] = -2E_{z_i}\left[(x_i - z_i) \cdot \frac{1}{N} \sum_{1}^{N} (x_\ell - z_i)\right]$$

Now since the resolution cells are independent, it follows that:

$$E_{z_i}\left[(x_i - z_i)(x_\ell - z_i)\right] = E_{z_i}(x_i - z_i) \, E_{z_i}(x_\ell - z_i) \text{ for } i = \ell.$$

but from (A.6b): $E_{z_i}(x_i - z_i) = 0$

Thus $E_{z_i}\left[(x_i - z_i)(x_\ell - z_i)\right] = 0 \quad$ for $i = \ell$.

and $E_{z_i}\left[(x_i - z_i)(x_\ell - z_i)\right] = E_{z_i}\left[(x_i - z_i)^2\right] \quad$ for $i = \ell$.

Finally, using (A.11):

$$E_{z_i}\left[(x_i - z_i)(x_\ell - z_i)\right] = \frac{1}{J} \sigma^2_{y/z_i} \qquad \text{for } i = \ell.$$

Substituting this back into the expression of the second term:

$$-2E_{z_i}\left[(x_i - z_i)(\bar{X} - z_i)\right] = -\frac{2}{NJ} \sigma^2_{y/z_i} \tag{A.12}$$

Finally, for the third term in (A.10).

$$E_{z_i}(\bar{X} - z_i)^2 = \frac{1}{N^2} E_{z_i}\left[\sum_{\ell=1}^{N}(x_\ell - z_i)^2\right]$$

As before, expanding the square of the summation, and taking the expectation inside the resulting double-summation:

$$E_{z_i}(\bar{X} - z_i)^2 = \frac{1}{N^2} \sum_{\ell}^{N} \sum_{k}^{N} E_{z_i}\left[(x_\ell - z_i)(x_k - z_i)\right] \qquad (A.13)$$

The expectation inside the double summation is evaluated individually for each of the following four cases:

Case A: $i \neq k \neq \ell$ and
Case B: $i = k \neq \ell$ and $i = \ell \neq k$
Case C: $i = k = \ell$
Case D: $i \neq k = \ell$

For Case A: $i \neq k \neq \ell$

$$E_{z_i}\left[(x_\ell - z_i)(x_k - z_i)\right] = (z_\ell - z_i)(z_k - z_i)$$

This case occurs $N^2-N-2(N-1)$ times in the summation of equation (A.13).

For Case B: $i = k \neq \ell$ and $i \neq \ell = k$

$$E_{z_i}\left[(x_\ell - z_i)(x_k - z_i)\right] = E_{z_i}\left[(x_i - z_i)(x_k - z_i)\right]$$

$$= E_{z_i}(x_i - z_i)E(x_k - z_i).$$

since x_i and x_k are independent.

But $E(x_i - z_i) = 0$

Thus $E_{z_i}(x_\ell - z_i)(x_k - z_i) = 0$

this case occurs 2(N-1) times in the summation of equation (A.13).

For case C: $i = k = \ell$

$$E_{z_i}\left[(x_\ell - z_i)(x_k - z_i)\right] = E_{z_i}\left[(x_i - z_i)^2\right] = \frac{1}{J}\sigma^2_{y/z_i}$$

where the last step follows from (A.11).

This case occurs only once in the summation of equation (A.13).

For case D: $i \neq (k=\ell)$

$$E_{z_i}\left[(x_\ell - z_i)(x_k - z_i)\right] = E_{z_i}\left[(x_\ell - z_i)^2\right] \qquad i \neq \ell$$

Since the indicated expectation is over frequency, expand the above as:

$$E_{z_i}\left[(x_\ell - z_i)^2\right] = E_{z_i}\left[(x_\ell - z_\ell) - (z_i - z_\ell)\right]^2$$

Using (A.11) and (A.6b), gives:

$$E_{z_i}\left[(x_\ell - z_i)^2\right] = \frac{1}{J}\sigma^2_{y/z_\ell} + (z_i - z_\ell)^2$$

This case occurs (N-1) times in the summation of equation (A.13).

Now, substituting the results from each of the above four cases into (A.13) gives:

$$E_{z_i}(M - z_i)^2 = \frac{1}{N^2} \underbrace{\sum_{\substack{k \\ k \neq \ell \neq i}}^{N} \sum_{\ell}^{N} (z_k - z_i)(z_\ell - z_i)}_{\text{(Case A)}}$$

$$+ \frac{1}{N^2} \underbrace{\sum_{\ell \neq i}^{N} \left[(z_\ell - z_i)^2 + \frac{1}{J} \sigma^2_{y/z_\ell} \right]}_{\text{(Case D)}} + \underbrace{\frac{1}{N^2 J} \sigma^2_{y/z_i}}_{\text{(Case C)}} \tag{A.14}$$

Now substituting (A.14), (A.12) and (A.11) into (A.10) gives:

$$E_{z_i}\left[(x_i - M)^2\right] = \frac{1}{J} \sigma^2_{y/z_i} - \frac{2}{NJ} \sigma^2_{y/z_i} + \frac{1}{N^2 J} \sigma^2_{y/z_i} + \frac{N-1}{N^2 J} \sigma^2_{y/z_i}$$

$$+ \frac{1}{N^2} \sum_{\substack{k \\ k \neq \ell \neq i}}^{N} \sum_{\ell}^{N} (z_k - z_i)(z_\ell - z_i) + \frac{1}{N^2} \sum_{\ell \neq i}^{N} (z_\ell - z_i)^2$$

Collecting terms and substituting into (A.9) gives:

$$E[V] = \frac{N-1}{N^2 J} \sum_{i=1}^{N} E\, \sigma^2_{y/z_i}$$

$$+ \frac{1}{N^3} \sum_{\substack{i \\ i \neq k \neq \ell}}^{N} \sum_{k}^{N} \sum_{\ell}^{N} E \left[(z_k - z_i)(z_\ell - z_i)\right] + \frac{1}{N^3} \sum_{i}^{N} \sum_{\ell \neq i}^{N} E \left[(z_\ell - z_i)^2\right] \tag{A.15}$$

Expanding the expectations in the last two summations of (A.15) gives:

$$\underset{i=k,\ell}{E}\ (z_k-z_i)(z_\ell-z_i) = E\{[(z_k-\mu_z) - (z_i-\mu_z)][(z_\ell-\mu_z)-(z_i-\mu_z)]\}$$

$$= E\ \{(z_k-\mu_z)(z_\ell-\mu_z)-(z_i-\mu_z)[(z_k-\mu_z)+(z_\ell-\mu_z)]+(z_i-\mu_z)^2\}$$

Since z_i is uncorrelated with z_k, z_ℓ for $i=k,\ell$ it follows that:

$$E\{(z_k - z_i)(z_\ell - z_i)\} = \begin{cases} \sigma_z^2 & \text{for } k\neq\ell\neq i \\ 2\sigma_z^2 & \text{for } k=\ell\neq i \end{cases} \qquad \text{(A.16)}$$

Now recall that the number of terms in the double sum over k,ℓ for $k=\ell=i$ in (A.15) is $N^2-N-2(N-1) = (N-1)(N-2)$, and that the number of terms in the last sum for $\ell=i$ is $(N-1)$. Therefore when the result of (A.16) is substituted intc the summation of (A.15) the following expression is obtained:

$$E[V] = \frac{N-1}{N^2J}\sum_{i=1}^{N} E\{\sigma_{y/z_i}^2\} + \frac{1}{N^3}\sum_{i=1}^{N}[(N-1)(N-2)\sigma_z^2 + 2(N-1)\sigma_z^2]$$

Finally, performing the indicated summation gives:

$$E[V] = \frac{N-1}{NJ}\ E\{\sigma_{y/z_i}^2\} + \frac{N-1}{N}\ \sigma_z^2$$

$$E[V] = \frac{N-1}{N}\left[\frac{1}{J}\ E\{\sigma_{y/z_i}^2\} + \sigma_z^2\right] \qquad \text{(A.17)}$$

Thus the expected value of the sample variance of the cell frequency-averages over the stencil, defined by equation (A.5) consists of two components as indicated in equation (A.17). The first component is due to the variance of the returns from any given cell over frequency, and this component decreases inversely as the number, J, of the frequencies averaged. The second component is the underlying variance, over space, of the frequency-averaged clutter returns from the different cells in the stencil. This second component depends upon the type of clutter - being low for "smooth" clutter such as flat, mown fields and high for "rough" clutter such as trees and mixtures of two or more different clutter types. Note that this second component is independent of the number of frequencies averaged, and effectively places a lower bound on the expected value of the sample variance, V.

Hence the effectiveness of frequency averaging depends upon the relative magnitudes of these two components. The larger the variance of the intrinsic spatial amplitude distribution of the clutter is in relation to that of the returns over frequency, the less effective frequency-averaging will be in reducing E[V].

The preceding results were derived without making any assumptions about the specific form of the underlying distributions p(y/z) and p(z). However in order to apply equation (A.17) to real data, an understanding of the actual distributions involved is helpful.

Specifically, it has been verified by Stovall[2] that homogeneous clutter (i.e., clutter of a given type - grass, trees, scrub, etc. - in contrast to mixed clutter) can be very well represented by a log-normal distribution whose mean and variance are determined by the specific clutter type in question, and furthermore that the distribution of the returns from a given resolution cell over frequency is approximately Rayleigh or exponential, depending upon whether the return is expressed in terms of voltage or power respectively. Furthermore, in order to provide the maximum possible dynamic range, logarithmic detectors are often used for amplitude CFAR systems of the type being analyzed. As a consequence of the logarithmic detection process, the distribution of the log-normally distributed clutter returns will appear normal, with a mean and variance that are directly expressible in dB (since the output of a log detector is linear in dB).

It is shown in Appendix C that the variance of the output of a log detector in response to an input signal that is Rayleigh or exponentially distributed is a constant regardless of the mean value of its input. As a result, when the frequency diverse returns y_{ij} from a given resolution cell (cell i) are obtained as outputs of a logarithmic detector, the conditional variance σ^2_{y/z_i} of the log detector outputs will be constant, independent of their mean value z_i.

APPENDIX B

Normality of the Probability Distribution of the Frequency Averages of Logarithmically Detected Clutter Returns

The analysis of clutter false alarm probability in section 3 was based upon the assumption that when the returns from log-normally distributed clutter are logarithmically detected, the frequency averages of the detector outputs will be normally distributed. The purpose of this appendix is to justify this assumption.

It has been shown [Stovall] that the amplitude of clutter returns can be characterized by a two-dimensional probability distribution over both radar frequency and space (resolution cells) in the following way:

$$p_{y,M_y}(y,M_y) = p_{y/M_y}(y/M_y)\ p_{M_y}(M_y)$$

where y is the amplitude of the clutter return from any resolution cell at any frequency and M_y is the mean value of the clutter return from a given resolution cell, averaged over frequency.

Thus $p(y/M_y)$ is the conditional distribution, over frequency, of the amplitude of clutter returns from <u>any given</u> resolution cell, for which the mean amplitude, over frequency, is M_y. Hence, M_y represents the intrinsic amplitude of the clutter within the given resolution cell. It has been postulated and experimentally verified that $p(y/M_y)$ is Rayleigh distributed, given the specified mean value. The distribution $p(M_y)$, on the other hand, is the distribution of the intrinsic clutter amplitude over

space. Hence it represents the distribution over space of the frequency averages of clutter returns from any given resolution cell. It has also been verified that $p_{M_y}(M_y)$ is log-normally distributed for a wide variety of clutter types.

This two-dimensional separation of the distribution of the clutter returns over both frequency and space is valid regardless of any intermediate functional transformation of the returns. Hence if the y's represent the output of a logarithmic detector, the previous argued separability of the two-dimensional distribution still holds, except that now the form of $p_{y/M_y}(y/M_y)$ will be logged-Rayleigh while that of $p_{M_y}(M_y)$, will be normal. However since the x_i's are sums of independent r.v.'s (the log-detector outputs y_{ij}) each of which are distributed in accordance with $p_{y/M_y}(y/M_y)$, it follows from the central limit theorem that for a sufficiently large number of frequencies averaged, the x_i's will all be approximately normally distributed. This is particularly true if the y_{ij}'s are logged-Rayleigh distributed as postulated, since that distribution has a shape similar to that of a normal distribution of like mean and variance. Because of the separability argument the distribution of the x_i's may again be written:

$$p(x,M_x) = p_{x/M_x}) \, p_{M_x}(M_x)$$

Furthermore, since the mean of the sum is equal to the sum of the means, it also follows the $M_x = M_y$. Thus the distribution of $p_{M_x}(M_x)$ will be the same as $p_{M_y}(M_y)$ - i.e., normal for the case of logarithmic detection of log-normally distributed clutter returns, while that of

$p_{x/M_x}(x/M_x)$ will be approximately normal by virtue of the averaging process used to form the x_i's and the central-limit theorem. Hence the density $p(x,M_x)$ is the product of two normal distributions and can be shown to be jointly normal. The r.v.'s x and M_x are of course <u>not</u> independent. The distribution of the x_i for all M_x is then given by the marginal density of $p(x,M_x)$ as follows:

$$p_x(x) = \int_{-\infty}^{+\infty} p(x,M_x)\, dM_x = \int_{-\infty}^{\infty} p_{x/M_x}(x/M_x)\, p_{M_x}(M_x)\, dM_x$$

Since $p(x,M_x)$ is jointly normal, the density $p_x(x)$ will also be normal. This completes the verification of the normality of the distribution of the x_i when log-normal clutter is logarithmically detected.

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APPENDIX C

Variance of the Output of a Logarithmic Detector in Response to a Rayleign Distributed Noise Input

Let v be the magnitude of the complex signal voltage at the input to a logarithmic detector. If v is a Rayleigh distributed random variable, then its probability density function is given by:

$$f_v(v) = \frac{v}{\sigma_x^2} e^{-\frac{v^2}{2\sigma_x^2}} \tag{C.1}$$

where σ_x^2 is the variance of the real and imaginary parts of the underlying complex Gaussian random process. For the present purposes it is more convenient to work with the instantaneous power corresponding to this input voltage, rather than with the voltage itself. Hence, define:

$$q = v^2 \tag{C.2}$$

The distribution of q is easily found to be [C-1]:

$$f_q(q) = \frac{e^{-q/\mu_q}}{\mu_q} \tag{C.3}$$

where $\mu_q = 2\sigma_x^2$, the mean value of v^2 – the mean input power. The mean value of the input voltage magnitude, $\overline{v}$, is related to μ_q by: $\overline{v} = \frac{\pi}{4}\sqrt{\mu_q}$ The output of a logarithmic detector can be calibrated to be linear in dB as follows: let x represent the voltage at the output of the logarithmic detector, then:

$$x = K \log_{10} v, \text{ volts} \tag{C.4}$$

where K is the log detector proportionality constant. However, the decibel level of the input signal, relative to an arbitrary 1 volt level, is defined by:

$$y = 20 \log_{10} v, \text{ dB} \tag{C.5}$$

$$\text{Thus } y = \frac{20}{K} x$$

In other words, multiplying the log-detector output voltage, x, by the calibration constant (20/K) yields the signal level in dB.

Substituting equation C.2 into C.5 gives:

$$y = 10 \log_{10} q$$

Furthermore, since $\log_{10} q = 0.4343 \text{ Ln } q$ it follows that:

$$y = 4.343 \text{ Ln } q \quad \text{dB} \tag{C.6}$$

We wish to determine the variance of y for any Rayleigh distributed input signal of arbitrary magnitude (mean power μ_q, or mean voltage $\frac{\pi}{4}\sqrt{\mu_q}$)

By definition:

$$\sigma_y^2 = E[y^2] - E^2[y] \tag{C.7}$$

Using the density function of q, given by equation C.3, and the definition of y given by C.6 it follows that:

$$E[y] = 4.343 \int_0^\infty (\text{Ln } q) \frac{e^{-q/\mu_q}}{\mu_q} dq \tag{C.8a}$$

$$E[y^2] = (4.343)^2 \int_0^\infty (\text{Ln } q)^2 \frac{e^{-q/\mu_q}}{\mu_q} dq \tag{C.8b}$$

The integrals in C.8a and C.8b are tabulated in Ref. C.2:

$$\int_0^\infty \text{Ln } q \frac{e^{-q/\mu_q}}{\mu_q} dq = -0.57721 + \text{Ln } \mu_q$$

$$\int_0^\infty (\text{Ln } q)^2 \frac{e^{-q/\mu_q}}{\mu_q} dq = (-0.57721 + \text{Ln } \mu_q)^2 + \frac{\pi^2}{6}$$

Thus $E[y] = 4.343 (-0.57721 + \text{Ln } \mu_q)$ (C.9a)

$$E[y^2] = (4.343)^2 [(-0.57721 + \text{Ln } \mu_q)^2 + \frac{\pi^2}{6}] \tag{C.9b}$$

Substituting these results into the definition of σ_y^2 equation C.7, gives:

$$\sigma_y^2 = (4.343)^2 \frac{\pi^2}{6} = 31.025 \text{ dB}^2$$

or $\sigma_y = 5.57$ dB

Note that this result is independent of the mean value of the Rayleigh distributed input signal.

References:

C.1 Burdic, William S., "Radar Signal Analysis," Prentice-Hall, Inc. 1968, p. 262.

C.2 Gradshteyn, I. S. and Ryzhik, I. M., "Tables of Integrals, Series and Products," Academic Press 1965, p. 573-574.

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Incoherent averaging of radar returns measured at different frequencies has been shown to be an effective method of improving fixed target detection performance. This improvement results from the fact that frequency-averaging narrows the amplitude probability distributions of both the target and clutter returns (after averaging), thereby allowing returns from targets to the more easily distinguished from those of clutter. This report examines the effect of frequency averaging on decreasing the variance of the estimated clutter reflectivity. The attainable reduction in clutter sample variance that results from frequency-averaging is shown to be strongly dependent upon the spatial "non-unformity" of the clutter. The more non-uniform the clutter, the less effective frequency-averaging is in reducing the clutter sample variance and corresponding CFAR threshold setting. An analytic expression is derived for the amount of sample variance reduction provided by frequency averaging, as a function of spatial standard deviation of mean clutter reflectivity and the number of frequencies averaged. The amount of variance reduction predicted by this expression is comfirmed by experimental measurements (at Ku band) of clutter returns, at 16 and 32 different frequencies, for a variety of different clutter types.